Appendix A: Confidence Intervals for State and Aggregate Emissions

Stability of the VARs implies that their dynamic forecasts of emission precursors are covariance stationary. We may therefore write the variance of the forecast growth rates of a particular state's CO_2 emissions in (9) as:

$$
\begin{aligned}
\sigma^2\left(G^\star_{T+k}\right) \approx\ & 2\left(1-\overline{\mu}\left[\log\widetilde{\Xi}^\star\right]\right)\sigma^2\left[\log\widetilde{\Xi}^\star_{T+k}\right] + 2\left(1-\overline{\mu}\left[\log\widetilde{\Phi}^\star\right]\right)\sigma^2\left[\log\widetilde{\Phi}^\star_{T+k}\right] \\
& + 2\left(1-\overline{\mu}\left[\log\widetilde{\Theta}^\star\right]\right)\sigma^2\left[\log\widetilde{\Theta}^\star_{T+k}\right] + 2\left(1-\overline{\mu}\left[\log\widetilde{\mathscr{I}}^\star\right]\right)\sigma^2\left[\log\widetilde{\mathscr{I}}^\star_{T+k}\right] \\
& + 2\left(1-\overline{\mu}\left[\log\widetilde{\mathscr{N}}^\star\right]\right)\sigma^2\left[\log\widetilde{\mathscr{N}}^\star_{T+k}\right] \\
& + 4\overline{\rho}\left[\Delta\log\widetilde{\Xi}^\star,\Delta\log\widetilde{\Phi}^\star\right]\sigma\left[\log\widetilde{\Xi}^\star_{T+k}\right]\sigma\left[\log\widetilde{\Phi}^\star_{T+k}\right]\sqrt{\left(1-\overline{\mu}\left[\log\widetilde{\Xi}^\star\right]\right)\left(1-\overline{\mu}\left[\log\widetilde{\Phi}^\star\right]\right)} \\
& + 4\overline{\rho}\left[\Delta\log\widetilde{\Xi}^\star,\Delta\log\widetilde{\Theta}^\star\right]\sigma\left[\log\widetilde{\Xi}^\star_{T+k}\right]\sigma\left[\log\widetilde{\Theta}^\star_{T+k}\right]\sqrt{\left(1-\overline{\mu}\left[\log\widetilde{\Xi}^\star\right]\right)\left(1-\overline{\mu}\left[\log\widetilde{\Theta}^\star\right]\right)} \\
& + 4\overline{\rho}\left[\Delta\log\widetilde{\Xi}^\star,\Delta\log\widetilde{\mathscr{I}}^\star\right]\sigma\left[\log\widetilde{\Xi}^\star_{T+k}\right]\sigma\left[\log\widetilde{\mathscr{I}}^\star_{T+k}\right]\sqrt{\left(1-\overline{\mu}\left[\log\widetilde{\Xi}^\star\right]\right)\left(1-\overline{\mu}\left[\log\widetilde{\mathscr{I}}^\star\right]\right)} \\
& + 4\overline{\rho}\left[\Delta\log\widetilde{\Xi}^\star,\Delta\log\widetilde{\mathscr{N}}^\star\right]\sigma\left[\log\widetilde{\Xi}^\star_{T+k}\right]\sigma\left[\log\widetilde{\mathscr{N}}^\star_{T+k}\right]\sqrt{\left(1-\overline{\mu}\left[\log\widetilde{\Xi}^\star\right]\right)\left(1-\overline{\mu}\left[\log\widetilde{\mathscr{N}}^\star\right]\right)} \\
& + 4\overline{\rho}\left[\Delta\log\widetilde{\Phi}^\star,\Delta\log\widetilde{\Theta}^\star\right]\sigma\left[\log\widetilde{\Phi}^\star_{T+k}\right]\sigma\left[\log\widetilde{\Theta}^\star_{T+k}\right]\sqrt{\left(1-\overline{\mu}\left[\log\widetilde{\Phi}^\star\right]\right)\left(1-\overline{\mu}\left[\log\widetilde{\Theta}^\star\right]\right)} \\
& + 4\overline{\rho}\left[\Delta\log\widetilde{\Phi}^\star,\Delta\log\widetilde{\mathscr{I}}^\star\right]\sigma\left[\log\widetilde{\Phi}^\star_{T+k}\right]\sigma\left[\log\widetilde{\mathscr{I}}^\star_{T+k}\right]\sqrt{\left(1-\overline{\mu}\left[\log\widetilde{\Phi}^\star\right]\right)\left(1-\overline{\mu}\left[\log\widetilde{\mathscr{I}}^\star\right]\right)} \\
& + 4\overline{\rho}\left[\Delta\log\widetilde{\Phi}^\star,\Delta\log\widetilde{\mathscr{N}}^\star\right]\sigma\left[\log\widetilde{\Phi}^\star_{T+k}\right]\sigma\left[\log\widetilde{\mathscr{N}}^\star_{T+k}\right]\sqrt{\left(1-\overline{\mu}\left[\log\widetilde{\Phi}^\star\right]\right)\left(1-\overline{\mu}\left[\log\widetilde{\mathscr{N}}^\star\right]\right)} \\
& + 4\overline{\rho}\left[\Delta\log\widetilde{\Theta}^\star,\Delta\log\widetilde{\mathscr{I}}^\star\right]\sigma\left[\log\widetilde{\Theta}^\star_{T+k}\right]\sigma\left[\log\widetilde{\mathscr{I}}^\star_{T+k}\right]\sqrt{\left(1-\overline{\mu}\left[\log\widetilde{\Theta}^\star\right]\right)\left(1-\overline{\mu}\left[\log\widetilde{\mathscr{I}}^\star\right]\right)} \\
& + 4\overline{\rho}\left[\Delta\log\widetilde{\Theta}^\star,\Delta\log\widetilde{\mathscr{N}}^\star\right]\sigma\left[\log\widetilde{\Theta}^\star_{T+k}\right]\sigma\left[\log\widetilde{\mathscr{N}}^\star_{T+k}\right]\sqrt{\left(1-\overline{\mu}\left[\log\widetilde{\Theta}^\star\right]\right)\left(1-\overline{\mu}\left[\log\widetilde{\mathscr{N}}^\star\right]\right)} \\
& + 4\overline{\rho}\left[\Delta\log\widetilde{\mathscr{I}}^\star,\Delta\log\widetilde{\mathscr{N}}^\star\right]\sigma\left[\log\widetilde{\mathscr{I}}^\star_{T+k}\right]\sigma\left[\log\widetilde{\mathscr{N}}^\star_{T+k}\right]\sqrt{\left(1-\overline{\mu}\left[\log\widetilde{\mathscr{I}}^\star\right]\right)\left(1-\overline{\mu}\left[\log\widetilde{\mathscr{N}}^\star\right]\right)}
\end{aligned}
$$

in which $\sigma[\cdot]$ and $\overline{\mu}[\cdot]$ denote each log index number's contemporaneous forecast standard error and long-run autocorrelation coefficient, and $\overline{\rho}[\cdot,\cdot]$ is the long-run correlation coefficient of the first-differences of any pair of log emission precursor series. The autocorrelation and inter-series correlation coefficients are both computed from the data rather than the VAR forecast trajectories of the variables. In turn, the delta method is used to compute the variance of the level of emissions in each state based on the recurrence relation derived from eq. (10):

$$\begin{aligned}\sigma^2\left[C^\star_{T+k}\right] &= \left(\frac{\partial C^\star_{T+k}}{\partial G^\star_{T+k}}\right)^2 \sigma^2\left[G^\star_{T+k}\right] + \left(\frac{\partial C^\star_{T+k}}{\partial C^\star_{T+k-1}}\right)^2 \sigma^2\left[C^\star_{T+k-1}\right] \\ &\quad + 2\,\frac{\partial C^\star_{T+k}}{\partial G^\star_{T+k}}\frac{\partial C^\star_{T+k}}{\partial G^\star_{T+k}}\,\rho\left[G^\star_{T+k}, C^\star_{T+k-1}\right]\sigma\left[G^\star_{T+k}\right]\sigma\left[C^\star_{T+k-1}\right] \\ &= 16\,\frac{(C^\star_{T+k-1})^2}{(2-G^\star_{T+k})^4}\,\sigma^2\left[G^\star_{T+k}\right] + \left(\frac{2+G^\star_{T+k}}{2-G^\star_{T+k}}\right)^2 \sigma^2\left[C^\star_{T+k-1}\right] \\ &\quad + 8\,\frac{C^\star_{T+k-1}(2+G^\star_{T+k})}{(2-G^\star_{T+k})^3}\,\rho\left[G^\star_{T+k}, C^\star_{T+k-1}\right]\sigma\left[G^\star_{T+k}\right]\sigma\left[C^\star_{T+k-1}\right]\end{aligned}$$

with the initial condition $\sigma\left[C^\star_{s,T}\right] = \sigma\left[C_{s,T}\right] = 0$. Our final step is to use the resulting sequence of state-level variances to construct the variance of emissions at the regional and aggregate levels as:

$$\sigma^2\left[\mathscr{C}^\star_{T+k}\right] = \sum_{s\in S}\sigma^2\left[C^\star_{s,T+k}\right] + 2\sum_{r\in S}\sum_{\substack{s\neq r\\ s\in S}}\overline{\zeta}\left[C^\star_r, C^\star_s\right]\sigma\left[C^\star_{r,T+k}\right]\sigma\left[C^\star_{s,T+k}\right].$$

where $\overline{\zeta}[\cdot,\cdot]$ is the historical correlation between the emission series of any pair of states r and s.